Section 1-1, Mathematics 104

We are going to review some basic mathematical skills which I believe you will need to succeed in this class.

Prime and Composite Numbers

The natural numbers are the numbers 1, 2, 3, ...

Each natural number is either a prime number or a composite number. A composite number is one that can be expressed as the product of two natural numbers other than 1. It has at least two other factors other than itself and 1.

A prime number is a natural number that has only two factors, 1 and itself.
Important skills are to be able to determine if a number is prime or composite and if composite, be able to express it as a product of prime factors.

The Fundamental Theorem of Arithmetic

The fundamental theorem of arithmetic says that a natural number can only be expressed as a product of prime numbers in exactly one unique way. It's important to know this since it means that there is only one right way to do this.

Divisibility Tests

2 - If the first digit is divisible by 2, meaning 0, 2, 4, 6, or 8, the number is divisible by 2
3 - Sum the digits. If necessary repeat until you have just 1 digit. If digit is divisible by 3, eg. 0, 3, 6 or 9, the original number is divisible by 3
4 - If the last two digits are divisible by 4, the number is divisible by 4
5 - If the last digit is 0 or 5, the number is divisible by 4
7 - Remove the last digit, multiply it by 2 and subtract from the number
Repeat until you have just one digit. If the digit is 0 or 7, the number is divisible by 7
8 - If the last three digits are divisible by 8, the number is divisible by 8 (see a pattern?
9 - Sum the digits. If necessary repeat until you have just 1 digit. If the digit is 0 or 9 the original number is divisible by 9
11 - Sum alternating digits. If they are equal or differ by 11, the original number is divisible by 11

Using these tests you can break down any number less than 121 into prime factors

Other important numbers

You should know the squares of numbers up to 100 and a little beyond

1	2	3	4	5	6	7	8	9	10
1	4	9	16	25	26	49	64	81	100

Also you should know the following squares

$11^2 = 121$

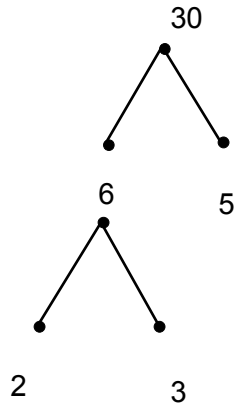
$12^2 = 144$

$15^2 = 225$

$16^2 = 256$

Prime factoring

Any number can be broken down into its prime factors. Sometimes a tree structure is useful for this. eg.



So $30 = 2 \times 3 \times 6$

It's helpful to recognize the prime numbers < 100

2, 3, 5, 7
11, 13, 17, 19
23, 29
31, 37
41, 43, 47
53, 59
61, 67
71, 73, 79
83, 89
97

There are only 25

Let's try factoring some numbers

72
78
84
43
95
68
133
145
717

Reducing Fraction by factoring

$$\frac{72}{87}=\frac{2\cdot 2\cdot 2\cdot 3\cdot 3}{3\cdot 29}=\frac{2\cdot 2\cdot 2\cdot 3}{29}=\frac{24}{29}$$

$$\frac{136}{150}=\frac{2\cdot 2\cdot 2\cdot 17}{2\cdot 3\cdot 5\cdot 5}=\frac{2\cdot 2\cdot 17}{3\cdot 5\cdot 5}=\frac{68}{75}$$

Let's try some

77/132
45/80
248/722

One way to think of this is in terms of the **GCM** or greatest common factor.
The GCM of two numbers is the greatest number that you can divide both by evenly.

Example

What is the GCM of 14 and 6?

14 = 2 x 7
6 = 2 x 3

So the GCM is 2

If you divide the numerator and denominator of a fraction by the GCM of these two numbers, a number will be reduced to lowest terms.

Adding, Subtracting, Multiplying and Dividing Fractions

Multiplying fractions is the easiest operation. You multiply numerator by numerator and denominator by denominator.

Example:

$$\frac{3}{14}\times\frac{5}{6}=\frac{3\cdot 5}{14\cdot 6}=\frac{15}{84}$$

Note that it can be useful to first factor and reduce

$$\frac{3}{14}\times\frac{5}{6}=\frac{3\cdot 5}{14\cdot 6}=\frac{3\cdot 5}{2\cdot 7\cdot 2\cdot 3}=\frac{5}{2\cdot 7\cdot 2}\cdot\frac{3}{3}=\frac{5}{2\cdot 7\cdot 2}=\frac{5}{28}$$

To divide one fraction by the other, you instead multiply by the reciprocal.

To get the reciprocal of a fraction, switch the numerator and the denominator.
To get the reciprocal of a natural number, write it as the number over 1 and then switch the numerator and denominator.

Example

$$\frac{3}{14}\div\frac{5}{6}=\frac{3}{14}\times\frac{6}{5}=\frac{3\cdot 2\cdot 3}{2\cdot 7\cdot 5}=\frac{3\cdot 3}{7\cdot 5}=\frac{9}{35}$$

To add or subtract fractions, the fractions have to have the same "common" denominator. There are two ways to go about this.

1) for the fractions $\frac{a}{b}+\frac{c}{d}$ use the denominator *bd*. This will always work.

Example:

If the fractions are $\frac{3}{14}+\frac{5}{6}$ the new denominator can be 6 x 14 = 84

$$\frac{3}{14}+\frac{5}{6}=\frac{3}{14}\cdot\frac{6}{6}+\frac{5}{6}\cdot\frac{14}{14}=\frac{18}{84}+\frac{70}{84}=\frac{88}{84}=\frac{22}{21}\cdot\frac{4}{4}=\frac{22}{21}$$

An alternative is to factor *b* and *d* and find the LCM (Lowest Common Multiple)
The LCM of two numbers is the smallest number that both can divide evenly.

Example:

Find the lowest common multiple of 14 and 6

14 = 2 x 7
6 = 2 x 3

So the LCM = 2 x 3 x 7 = 42

$$\frac{3}{14}+\frac{5}{6}=\frac{3}{14}\cdot\frac{3}{3}+\frac{5}{6}\cdot\frac{7}{7}=\frac{9}{42}+\frac{35}{42}=\frac{44}{42}=\frac{22}{21}\cdot\frac{2}{2}=\frac{22}{21}$$